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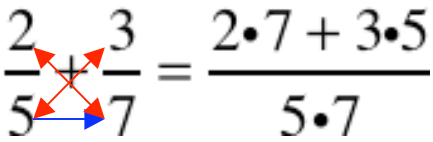
Situation 1 : Adding Fractions

Prompt (given by Kim Johnson)

A teacher asked her students to add:

$$\frac{2}{5}+\frac{3}{7}$$

One student in the class asked about the "cross-multiplication technique" he used to apply in this situation:



"To write the fraction with the denominator equals to the product of the two denominators, and the numerator equals to the sum of the "cross-products" of the numerators and denominators of the both fractions. Student asked: "Does it always work?"

Commentary

The foci consider several important aspects of adding the fractions. Focus 1 looks at adding the fractions with the Common Denominator. Focus 2 examines adding the fractions by using the Least Common Denominator.

Mathematical Foci

Mathematical Focus 1

One method for adding fractions is to rewrite the fractions as equivalent fractions with the common denominator.

One way to add the fractions $\frac{a}{b}+\frac{c}{d}$ $(b\neq 0, d\neq 0)$ is to find the common denominator by multiplying the denominators (bd). One has to rewrite the fractions as equivalent fractions with the common denominator: $\frac{a}{b}=\frac{a\bullet d}{b\bullet d}$, $\frac{c}{d}=\frac{c\bullet b}{d\bullet b}$.

Then one can add like fractions by adding the numerators and writing the sum over the common denominator:

$$\frac{a \bullet d}{b \bullet d} + \frac{c \bullet b}{d \bullet b} = \frac{a \bullet d + c \bullet b}{b \bullet d}$$

This method is corresponding to the students' "cross-multiply " technique to find the answer. Therefore, the answer to the students' question is "yes" .

Mathematical Focus 2

One method for adding fractions is to rewrite the fractions as equivalent fractions with the Least Common Denominator.

One can add the fractions by finding the smallest multiple (LCM) of both denominators and rewrite the fractions as equivalent fractions with the LCM as the denominator.

Sometimes the product of the two denominators will not be the lowest common denominator.

Both methods, after the reducing the answers to the simplest (lowest term), will lead to the same answer. However, in case the product of two denominators does not equal to the lowest common denominator, the procedure of reducing the answer to the lowest term will always take place.

Let add fractions $\frac{a}{k \bullet m} + \frac{c}{k \bullet n}$

Let the Least Common Denominator would be $k \bullet m \bullet n$. However, the product of the two denominators would be $k \bullet k \bullet m \bullet n$.

Therefore, in case one follows the first method, he will get

$$\frac{a}{k \bullet m} + \frac{c}{k \bullet n} = \frac{a \bullet k \bullet n}{k \bullet m} + \frac{c \bullet k \bullet m}{k \bullet n} = \frac{k(a \bullet n + c \bullet m)}{k \bullet k \bullet n \bullet m} = \frac{a \bullet n + c \bullet m}{k \bullet n \bullet m}$$